OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE UNIT

4724/01

MATHEMATICS

Core Mathematics 4

TUESDAY 23 JANUARY 2007

Afternoon

Time: 1 hour 30 minutes

Additional Materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

[Turn over

1 It is given that

$$f(x) = \frac{x^2 + 2x - 24}{x^2 - 4x} \quad \text{for } x \neq 0,\ x \neq 4.$$

Express $f(x)$ in its simplest form. [3]

2 Find the exact value of $\int_1^2 x \ln x \, dx$. [5]

3 The points A and B have position vectors **a** and **b** relative to an origin O, where $\mathbf{a} = 4\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$ and $\mathbf{b} = -7\mathbf{i} + 5\mathbf{j} + 4\mathbf{k}$.

(i) Find the length of AB. [3]

(ii) Use a scalar product to find angle OAB. [3]

4 Use the substitution $u = 2x - 5$ to show that $\int_{\frac{5}{2}}^{3} (4x - 8)(2x - 5)^7 \, dx = \frac{17}{72}$. [5]

5 (i) Expand $(1 - 3x)^{-\frac{1}{3}}$ in ascending powers of x, up to and including the term in x^3. [4]

(ii) Hence find the coefficient of x^3 in the expansion of $\left(1 - 3(x + x^3)\right)^{-\frac{1}{3}}$. [3]

6 (i) Express $\frac{2x + 1}{(x - 3)^2}$ in the form $\frac{A}{x - 3} + \frac{B}{(x - 3)^2}$, where A and B are constants. [3]

(ii) Hence find the exact value of $\int_4^{10} \frac{2x + 1}{(x - 3)^2} \, dx$, giving your answer in the form $a + b \ln c$, where a, b and c are integers. [4]

7 The equation of a curve is $2x^2 + xy + y^2 = 14$. Show that there are two stationary points on the curve and find their coordinates. [8]

8 The parametric equations of a curve are $x = 2t^2$, $y = 4t$. Two points on the curve are $P\,(2p^2, 4p)$ and $Q\,(2q^2, 4q)$.

(i) Show that the gradient of the normal to the curve at P is $-p$. [2]

(ii) Show that the gradient of the chord joining the points P and Q is $\frac{2}{p + q}$. [2]

(iii) The chord PQ is the normal to the curve at P. Show that $p^2 + pq + 2 = 0$. [2]

(iv) The normal at the point $R\,(8, 8)$ meets the curve again at S. The normal at S meets the curve again at T. Find the coordinates of T. [4]

9 **(i)** Find the general solution of the differential equation

$$\frac{\sec^2 y}{\cos^2(2x)} \frac{dy}{dx} = 2.$$ [7]

(ii) For the particular solution in which $y = \frac{1}{4}\pi$ when $x = 0$, find the value of y when $x = \frac{1}{6}\pi$. [3]

10 The position vectors of the points P and Q with respect to an origin O are $5\mathbf{i} + 2\mathbf{j} - 9\mathbf{k}$ and $4\mathbf{i} + 4\mathbf{j} - 6\mathbf{k}$ respectively.

(i) Find a vector equation for the line PQ. [2]

The position vector of the point T is $\mathbf{i} + 2\mathbf{j} - \mathbf{k}$.

(ii) Write down a vector equation for the line OT and show that OT is perpendicular to PQ. [4]

It is given that OT intersects PQ.

(iii) Find the position vector of the point of intersection of OT and PQ. [3]

(iv) Hence find the perpendicular distance from O to PQ, giving your answer in an exact form. [2]

1	Factorise numerator and denominator	M1		or Attempt long division
	Num = $(x+6)(x-4)$ or denom = $x(x-4)$	A1		Result = $1+\frac{6x-24}{x^2-4x}$
	Final answer = $\frac{x+6}{x}$ or $1+\frac{6}{x}$	A1	3	$= 1+\frac{6}{x}$
2	Use parts with $u = \ln x, \mathrm{d}v = x$	M1		& give 1st stage in form $\mathrm{f}(x) +/- \int \mathrm{g}(x)(\mathrm{d}x)$
	Obtain $\frac{1}{2}x^2 \ln x - \int \frac{1}{x}.\frac{1}{2}x^2 (\mathrm{d}x)$	A1		or $\frac{1}{2}x^2 \ln x - \int \frac{1}{2}x(\mathrm{d}x)$
	$= \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2$ (+c)	A1		
	Use limits correctly	M1		
	Exact answer $2 \ln 2 - \frac{3}{4}$	A1	5	AEF ISW
3	(i) Find $\boldsymbol{a}-\boldsymbol{b}$ or $\boldsymbol{b}-\boldsymbol{a}$ irrespective of label	M1		(expect $11\mathbf{i} - 2\mathbf{j} - 6\mathbf{k}$ or $-11\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}$)
	Method for magnitude of any vector	M1		
	$\sqrt{161}$ or $12.7(12.688578)$	A1	3	
	(ii) Using $(\overline{\mathrm{AO}} \text{ or } \overline{\mathrm{OA}})$ and $(\overline{AB} \text{ or } \overline{\mathrm{BA}})$	B1		Do not class angle *AOB* as MR
	$\cos\theta = \frac{\text{scalar product of any two vectors}}{\text{product of their moduli}}$	M1		
	43 or better (42.967…), 0.75 or better (0.7499218..)	A1	3	If 137 obtained, followed by 43, award A0 Common answer 114 probably → B0 M1 A0
4	Attempt to connect $\mathrm{d}x$ and $\mathrm{d}u$	M1		but not just $\mathrm{d}x = \mathrm{d}u$
	For $\mathrm{d}u = 2\,\mathrm{d}x$ AEF correctly used	A1		sight of $\frac{1}{2}$ ($\mathrm{d}u$) necessary
	$\int u^8 + u^7 (\mathrm{d}u)$	A1		or $\int u^7(u+1)(\mathrm{d}u)$
	Attempt new limits for u at any stage (expect 0,1)	M1		or re-substitute & use ($\frac{5}{2}$,3)
	$\frac{17}{72}$	A1	5	**AG** WWW
	S.R. If M1 A0 A0 M1 A0, award S.R. B1 for answer $\frac{68}{72}, \frac{34}{36}$ or $\frac{17}{18}$			ISW
5	(i) Show clear knowledge of binomial expansion	M1		$-3x$ should appear but brackets can be missing; $-\frac{1}{3}.-\frac{4}{3}$ should appear, not $-\frac{1}{3}.\frac{2}{3}$
	$= 1+x$	B1		Correct first 2 terms; not dep on M1
	$+2x^2$	A1		
	$+\frac{14}{3}x^3$	A1	4	
	(ii) Attempt to substitute $x+x^3$ for x in (i)	M1		Not just in the $\frac{14}{3}x^3$ term
	Clear indication that $(x+x^3)^2$ has no term in x^3	A1		
	$\frac{17}{3}$	√A1	3	f.t. $\mathrm{cf}(x) + \mathrm{cf}(x^3)$ in part (i)
6	(i) $2x+1 = / \equiv A(x-3)+B$	M1		
	$A = 2$	A1		
	$B = 7$	A/B 1	3	Cover-up rule acceptable for B1
	(ii) $\int \frac{1}{x-3}(\mathrm{d}x) = \ln(x-3)$ or $\ln\lvert x-3\rvert$	B1		Accept *A* or $\frac{1}{A}$ as a multiplier
	$\int \frac{1}{(x-3)^2}(\mathrm{d}x) = -\frac{1}{x-3}$	B1		Accept *B* or $\frac{1}{B}$ as a multiplier
	6 + 2 ln 7 Follow-through $\frac{6}{7}B + A\ln 7$	√B2	4	

7	$\frac{d}{dx}(xy) = x\frac{dy}{dx} + y$	B1		
	$\frac{d}{dx}(y^2) = 2y\frac{dy}{dx}$	B1		
	$4x + x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0$	B1		
	Put $\frac{dy}{dx} = 0$	*M1		
	Obtain $4x + y = 0$ AEF	A1		and no other (different) result
	Attempt to solve simultaneously with eqn of curve	dep*M1		
	Obtain $x^2 = 1$ or $y^2 = 16$ from $4x + y = 0$	A1		
	$(1,-4)$ and $(-1,4)$ and no other solutions	A1	8	Accept $(\pm 1, \mp 4)$ but not $(\pm 1, \pm 4)$
8	(i) Use $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ and $-\frac{1}{m}$ for grad of normal	M1		or change to cartesian.,diff & use $-\frac{1}{m}$
	$= -p$ **AG** WWW	A1	2	Not $-t$.
	(ii) Use correct formula to find gradient of line	M1		
	Obtain $\frac{2}{p+q}$ **AG** WWW	A1	2	Minimum of denom = $2(p-q)(p+q)$
	(iii) State $-p = \frac{2}{p+q}$	M1		Or find eqn normal at P & subst $(2q^2, 4q)$
	Simplify to $p^2 + pq + 2 = 0$ **AG** WWW	A1	2	With sufficient evidence
	(iv) (8,8) $\rightarrow t$ or p or $q = 2$ only	B1		No possibility of -2
	Subst $p = 2$ in eqn (iii) to find q_1	M1		Or eqn normal,solve simult with cartes/param
	Subst $p = q_1$ in eqn (iii) to find q_2	M1		Ditto
	$q_2 = \frac{11}{3} \rightarrow \left(\frac{242}{9}, \frac{44}{3}\right)$	A1	4	No follow-through; accept (26.9 , 14.7)
9	(i) Separate variables as $\int \sec^2 y \, dy = 2\int \cos^2 2x \, dx$	M1		seen or implied
	LHS = tan y	A1		
	RHS; attempt to change to double angle	M1		
	Correctly shown as 1 + cos 4x	A1		
	$\int \cos 4x \, dx = \frac{1}{4}\sin 4x$	A1		
	Completely correct equation (other than +c)	A1		$\tan y = x + \frac{1}{4}\sin 4x$
	+c on either side	A1	7	<u>not</u> on both sides unless c_1 and c_2
	(ii) Use boundary condition	M1		provided a sensible outcome would ensue
	c (on RHS) = 1	A1		or $c_2 - c_1 = 1$; not fortuitously obtained
	Substitute $x = \frac{1}{6}\pi$ into their eqn, produce y = 1.05	A1	3	or 4.19 or 7.33 etc. Radians only
10	(i) For (either point) + t(diff between posn vectors)	M1		"**r** =" not necessary for the M mark
	r = (either point) + t(**i** -2**j** – 3**k** or –**i** + 2**j** + 3**k**)	A1	2	**…** but it is essential for the A mark
	(ii) **r** = s(**i** + 2**j** – **k**) or (**i** + 2**j** – **k**) + s(**i** + 2**j** – **k**)	B1		Accept any parameter, including t
	Eval scalar product of **i**+2**j**-**k** & their dir vect in (i)	M1		
	Show as (1x1 or 1)+(2x-2 or -4)+(-1x-3 or 3)	A1		This is just one example of numbers involved
	= 0 <u>and</u> state perpendicular **AG**	A1	4	
	(iii) For at least two equations with diff parameters	M1		e.g. $5 + t = s$, $2 - 2t = 2s$, $-9 - 3t = -s$
	Obtain $t = -2$ or $s = 3$ (possibly -3 or 2 or -2)	A1		Check if $t = 2, 1$ or -1
	Subst. into eqn AB or OT and produce 3**i** + 6**j** – 3**k**	A1	3	
	(iv) Indicate that $\lvert OC \rvert$ is to be found	M1		where C is their point of intersection
	$\sqrt{54}$;f.t. $\sqrt{a^2 + b^2 + c^2}$ from a**i** + b**j** + c**k** in (iii)	√A1	2	

<u>In the above question, accept any vectorial notation</u>
t and s may be interchanged, and values stated above need to be treated with caution.
In (iii), if the point of intersection is correct, it is more than likely that the whole part is correct – but check.